Exoplanet transit parameters from amateur-astronomers observations

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1 Statement of the problem

Signs of exoplanets transiting their parent stars have been observed for a couple of years. While catching a giant planet transit is fairly easy even with amateur equipment, one would like to go down to Earth-like planets. One of the ways to achieve this goal focuses on detecting secular changes of the orbit of a giant planet caused by gravitational influence of another planet in the system. From the observational viewpoint the changes of the orbit manifest through variations in the mid-transit times, and depth and length of the transits.

This is similar to classical eclipsing binaries where the mid-eclipse timings can provide evidence of additional bodies in the system, apsidal motion or possibly magnetic activity in the system. Amateur astronomers have greatly contributed to the astrophysics of binary systems by long-term monitoring of eclipse timings. Nowadays the effort is being extended from the eclipsing binaries towards the exoplanet transits.

The methods that have been used to derive mid-eclipse times of eclipsing binaries haven't evolved much over the last decades. One of the most popular methods dates back to 1950s to the paper of Kwee and van Woerden (1956) and exploits the symmetrical shape of the eclipse light curve. This method isn't particularly suitable for derivation of mid-transit times because the ingress and egress parts of the light curves are very short and the signal-to-noise ratio is usually quite small. Further, amateur observations often suffer from systematics originating from airmass changes during the observing run that make the raw light curve asymetrical. Further, Kwee and van Woerden (1956) method does not yield transit depth a length.

Bruce Gary and his Amateur Exoplanet Archive[1] use a symmetric piecewise linear function to determine not only the time of mid-transit but also the depth, duration of partial and total eclipse and systematic trend and airmass curvature. However, a piecewise linear function is only a proxy to the real transit shape. Whether the difference from the true transit shape is significant in amateur data is a matter of debate.

For the purpose of the Exoplanet Transit Database[2] we've taken a different approach. Our requirement was a fast, reliable, reasonably precise and automatic procedure that can be brought to the observers through a web interface.

2 Fitting transit parameters

2.1 Variables and equations

We assume that the observations of the transit consist of N relative magnitudes m_i taken at times t_i and the photometry software provided a measurement error σ_i computed most likely from Poisson statistics

[1] http://brucegary.net/AXA/x.htm
[2] http://var2.astro.cz/tresca/transits.php

and read-out noise. We model the dataset by a function

$$m(t_i) = A - 2.5 \log F(z[t_i, t_0, D, b], p, c_1) + B(t_i - t_{\text{mean}}) + C(t_i - t_{\text{mean}})^2, \quad (1)$$

where $F(z, p, c_1)$ is a relative flux decrease from the star due to the transiting planet. We assume that the star and planet are limb darkened and dark disks, respectively, with radius ratio $p = R_{\text{planet}}/R_*$ and that the planet is much smaller than the star, $p \lesssim 0.2$. The projected relative separation of the planet from the star is z. Limb darkening of the star is modelled by the linear law with coefficient c_1. We employ the `occultsmall` routine of Mandel & Agol (2002) as our $F(z, p, c_1)$. We checked that the small planet approximation, $p \lesssim 0.2$, does not produce significant differences from the full model (at least for the typical values of p and having in mind the quality of the photometry) and is much faster to compute.

We model the planet trajectory as a straight line over the stellar disk with impact parameter $b = a \cos i/R_*$. The closest approach occurs at t_0 and the whole transit lasts D. From these assumptions we can compute $z[t_i, t_0, D, b]$ for every t_i.

Variables A, B and C in the equation (1) describe systematic trends in the data and the zero-point shift of the magnitudes. Linear and quadratic terms are computed with respect to the mean time of observations $t_{\text{mean}} = \sum_i t_i/N$ to supress numeric errors. We do not employ any explicit correction for airmass curvature as we think a generic second-degree polynomial is sufficient in most cases. There is no problem of adding higher-order terms in $(t_i - t_{\text{mean}})$, if the need ever arises.

2.2 Fiting procedure

We have used the Levenberg-Marquardt non-linear least squares fitting algorithm from Price et al. (1992). The algorithm requires initial values of parameters and partial derivatives of the fitted function. We take the initial values from literature (except for c_1, see below). We compute all partial derivatives of equation (1) analytically, except for $\partial F/\partial z$, $\partial F/\partial p$ and $\partial F/\partial c_1$ which were computed numerically using Ridders' method (procedure `dfridr` of Price et al. 1992).

The search for optimal parameters is done by iterating the fitting procedure until χ^2 does not change. Usually, with good initial values, about ten iterations are sufficient. We then rescale the error bars σ_i to make the final $\chi^2 = N - g$, where g is a number of free parameters (degrees of freedom), and we re-run the fitting procedure to obtain final errors of the parameters. Original photometric errors are usually underestimated and this procedure yields more reasonable errors of the output parameters.

In the optimal circumstances, one would regard as free parameters all variables in equation (1), namely A, B, C, t_0, D, b, p and c_1. However, these parameters are correlated to some extent and noisy photometry from small amateur telescopes just does not permit recovery of all of them. We need to fit zero-point shift A and in most cases also a linear systematic trend B. We didn't find any datasets in our database where setting C as a free parameter would improve the fit significantly. Our primary goal is to get the mid-transit time t_0, duration D and depth. Hence, we set t_0 and D as free parameters, by default. However, for a limb-darkened star, the depth of a transit is determined by planet radius p, impact factor b and limb darkening coefficient c_1. Primarily, the depth of the transit is governed by the planet radius p and we set it as free parameter. Parameters b and c_1 affect the depth and shape of the transit to a lesser extent and from noisy amateur data we couldn't retrieve meaningful values for the two parameters simultaneuosly with p. Therefore, we hold b and c_1 fixed. We either compute b from orbital parameters of the planet and radius of the star or take the value from the literature. The situation is more complicated for limb darkening because c_1 should be different for every photometric filter. We've decided to keep c_1 fixed at a rather arbitrary value $c_1 = 0.5$ in all cases. We've experimented with values from 0.2 to 0.9 and found that the effect on other parameters is rather negligable, usually smaller than the error bars. The export value of the depth is then evaluated as $-2.5 \log \min_z F(z, p, c_1)$.

References

[1] Kwee, K. K., van Woerden, H. 1956, Bull. of the Astr. Inst. of the Netherlands, 464, 327

[2] Mandel, K., Agol, E. 2002, ApJ, 580, 171

[3] Price, W. H., et al. 1992, Numerical Recipes in C